

Strongly Intensive Quantities

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Abstract

Analysis of fluctuations of hadron production properties in collisions of relativistic particles profits from use of measurable intensive quantities which are independent of system size variations. The first family of such quantities was proposed already in 1992. The second is introduced in this paper. We also present a proof of independence of volume fluctuations for quantities from both families within the framework of the grand canonical ensemble. These quantities are referred to as strongly intensive ones. Influence of conservation laws and resonance decays is also discussed.

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I. INTRODUCTION

An intensive quantity is a physical quantity which does not depend on the system volume. By contrast, an extensive quantity is proportional to the system volume. Clearly, the ratio of two extensive quantities is an intensive one. For example the number of particles, N , in the relativistic gas fluctuates around its mean value, $\langle N \rangle$. Within the grand canonical ensemble $\langle N \rangle$ is an extensive quantity, whereas the ratio of mean multiplicities of two different particle types is an intensive one. Particle number fluctuations are quantified by the variance, $\langle N^2 \rangle - \langle N \rangle^2$, which is an extensive quantity, while the scaled variance, $[\langle N^2 \rangle - \langle N \rangle^2]/\langle N \rangle$, is an intensive one.

Statistical models are surprisingly successful in modeling multi-particle production in high energy interactions [1]. They are used to describe properties of strongly interacting matter created in nucleus-nucleus collisions in terms of intensive quantities. In particular, an equation of state is usually given as a function relating pressure, temperature and baryonic chemical potential. On the other hand, in high energy collisions the volume of produced matter cannot be kept fixed. For instance, nucleus-nucleus collisions with different centralities may produce a statistical system with the same local properties (e.g., the same temperature and baryonic chemical potential) but with the system volume changing significantly from interaction to interaction. Thus, an important question is whether it is possible to measure the properties of the system without knowing its volume fluctuations, or equivalently, whether there are measurable quantities which are independent of volume fluctuations.

Within the grand canonical ensemble the answer is yes, and the quantities with the required properties will be referred to as strongly intensive ones. Ratios of mean particle multiplicities are strongly intensive quantities. In general, this is the case for all ratios of any two extensive quantities which correspond to the first moments of fluctuating variables. They are intensive and strongly intensive quantities. The situation is, however, more complicated for the measures of fluctuations which include the second moments of fluctuating variables. For example, as will be shown below, the scaled variance of a particle number distribution is an intensive quantity, but not a strongly intensive one.

In this paper we show that there are two families of strongly intensive quantities which characterize the second moments of random extensive variables used to study fluctuations and correlations in a physical system. While the first family was introduced already in 1992 [2], the second one is proposed in this paper.

The paper is organized as follows. In Sec. II the two families of strongly intensive quantities

are introduced. For simplicity, this is done within the model of independent particle sources. The relation of the strongly intensive quantities to previously used fluctuation measures is discussed in Sec. III. The proof that the quantities are in fact strongly intensive, i.e., strictly independent of volume and volume fluctuations within the grand canonical ensemble is given in Sec. IV. Finally, their properties within the canonical and micro-canonical ensembles are discussed in Sec. V. Summary given in Sec. VI closes the paper.

II. TWO FAMILIES OF STRONGLY INTENSIVE QUANTITIES

Let us start from the model of independent sources for multi-particle production in which the number of sources, N_s , changes from event to event. In this model, extensive quantities (e.g., mean number of particles, mean transverse energy) will be considered as those which are proportional to N_s . Two fluctuating extensive variables A and B can be expressed as:

$$A = a_1 + a_2 + \dots + a_{N_s}, \quad B = b_1 + b_2 + \dots + b_{N_s}, \quad (1)$$

where a_k and b_k denote the contributions from the k -th source. One finds for event averages:

$$\langle A \rangle = \langle a \rangle \langle N_s \rangle, \quad \langle A^2 \rangle = \langle a^2 \rangle \langle N_s \rangle + \langle a \rangle^2 [\langle N_s^2 \rangle - \langle N_s \rangle], \quad (2)$$

$$\langle B \rangle = \langle b \rangle \langle N_s \rangle, \quad \langle B^2 \rangle = \langle b^2 \rangle \langle N_s \rangle + \langle b \rangle^2 [\langle N_s^2 \rangle - \langle N_s \rangle], \quad (3)$$

$$\langle AB \rangle = \langle a b \rangle \langle N_s \rangle + \langle a \rangle \langle b \rangle [\langle N_s^2 \rangle - \langle N_s \rangle], \quad (4)$$

where $\langle a \rangle$, $\langle b \rangle$ and $\langle a^2 \rangle$, $\langle b^2 \rangle$, $\langle ab \rangle$ are the first and second moments of the distribution $P^*(a, b)$ for a single source. These quantities are independent of N_s and play the role of intensive quantities in the model of independent sources. The distribution $P^*(a, b)$ is assumed to be the same for all sources, i.e., they are statistically identical. The probability distribution $P_s(N_s)$ of the source number is needed to calculate $\langle N_s \rangle$ and $\langle N_s^2 \rangle$ and, in general, it is unknown. Using Eq. (2) the scaled variance ω_A which describes the event-by-event fluctuations of the extensive variable A can be presented as:

$$\omega_A \equiv \frac{\langle A^2 \rangle - \langle A \rangle^2}{\langle A \rangle} = \frac{\langle a^2 \rangle - \langle a \rangle^2}{\langle a \rangle} + \langle a \rangle \frac{\langle N_s^2 \rangle - \langle N_s \rangle^2}{\langle N_s \rangle} \equiv \omega_a^* + \langle a \rangle \omega_s, \quad (5)$$

where ω_a^* is the scaled variance of the quantity A for each source. A similar expression follows from Eq. (3) for the scaled variance ω_B . The scaled variances ω_A and ω_B are independent of the average number of sources $\langle N_s \rangle$. Thus, ω_A and ω_B are intensive quantities. However, they depend on the fluctuations of the number of sources via ω_s and, therefore, they are not strongly intensive quantities.

From Eq. (2) follows that a knowledge of $\langle A \rangle$ and $\langle A^2 \rangle$ is not sufficient to derive any strongly intensive quantity. This is, however, possible when two extensive random variables, A and B are considered. In order to characterize fluctuations, one may then construct special combinations of the second moments in which the terms proportional to $\langle N_s^2 \rangle$ in the r.h.s. of Eqs. (2-4) are not present. Clearly, only two linearly independent combinations of this type result from three equations (2-4). Note that in order to remove the dependence on $\langle N_s \rangle$, strongly intensive quantities should be in a form of reducible fractions. The following combinations seem the most convenient:

$$\Sigma^{AB} = \langle C \rangle^{-1} [\langle B \rangle \omega_A + \langle A \rangle \omega_B - 2 (\langle AB \rangle - \langle A \rangle \langle B \rangle)] , \quad (6)$$

$$\Delta^{AB} = \langle C \rangle^{-1} [\langle B \rangle \omega_A - \langle A \rangle \omega_B] , \quad (7)$$

where $\langle C \rangle$ is the average of any extensive quantity, e.g., $\langle A \rangle$ or $\langle B \rangle$. Straightforward calculation of (6) and (7) using Eqs. (2-5) gives:

$$\Sigma^{AB} = \langle c \rangle^{-1} [\langle b \rangle \omega_a^* + \langle a \rangle \omega_b^* - 2 (\langle ab \rangle - \langle a \rangle \langle b \rangle)] , \quad (8)$$

$$\Delta^{AB} = \langle c \rangle^{-1} [\langle b \rangle \omega_a^* - \langle a \rangle \omega_b^*] . \quad (9)$$

Thus, Σ^{AB} and Δ^{AB} defined by Eqs. (6-7) depend on ω_a^* and ω_b^* , but they are independent of the average number of sources $\langle N_s \rangle$ and its fluctuations ω_s . They are, in fact, strongly intensive measures which quantify fluctuations of any two extensive random variables A and B . This will be proved in Sec. IV within the grand canonical ensemble for a case of particle multiplicities.

There is an important difference between the Σ^{AB} and Δ^{AB} quantities. Namely, in order to calculate Δ^{AB} one needs to measure only the first two moments: $\langle A \rangle$, $\langle B \rangle$ and $\langle A^2 \rangle$, $\langle B^2 \rangle$. This can be done by independent measurements of the distributions $P_A(A)$ and $P_B(B)$. Quantity Σ^{AB} includes the correlations term, $\langle AB \rangle - \langle A \rangle \langle B \rangle$, and thus it requires, in addition, simultaneous measurements of A and B in order to obtain the joint probability distribution $P_{AB}(A, B)$. The quantities Σ^{AB} and Δ^{AB} also have different symmetry properties: $\Sigma^{AB} = \Sigma^{BA}$ and $\Delta^{AB} = -\Delta^{BA}$. We call all strongly intensive quantities which include the correlation term the Σ family, those which include only variances the Δ family.

III. RELATION TO OTHER FLUCTUATION MEASURES

The well-known fluctuation measure Φ was introduced in 1992 [2] for a study of transverse momentum fluctuations and it belongs to the Σ family. In the general case [3], when $A \equiv X =$

$\sum_{i=1}^N x_i$ represents any motional extensive variable as a sum of single particle variables, and $B \equiv N$ is the particle multiplicity, one gets:

$$\Phi_x = \left[\frac{\langle X \rangle}{\langle N \rangle} \Sigma^{XN} \right]^{1/2} - \left[\overline{x^2} - \bar{x}^2 \right]^{1/2}, \quad (10)$$

where Σ^{XN} is given by Eq. (6) with $C \equiv N$, and $\overline{x^2}$, \bar{x}^2 correspond to single-particle inclusive averages. Note that these inclusive quantities can also be presented in terms of event averages, namely $\bar{x} = \langle X \rangle / \langle N \rangle$ and $\overline{x^2} = \langle X_2 \rangle / \langle N \rangle$, where $X_2 \equiv \sum_{i=1}^N x_i^2$. The measure Φ was extended in 1999 [4, 5] for multiplicity fluctuations. For two particle types, A and B , the Φ measure was constructed by setting $x_i = 1$ if the i -th particle is of the A type and $x_i = 0$ otherwise. One then finds $\bar{x} = \overline{x^2} = \langle A \rangle / [\langle A \rangle + \langle B \rangle]$ and thus $\overline{x^2} - \bar{x}^2 = \langle A \rangle \langle B \rangle / [\langle A \rangle + \langle B \rangle]^2$ with A and B denoting particle numbers. Taking into account these relations, and using $X = A$ and $N = A + B$ in Eq. (10), the expression for the Φ reads:

$$\Phi = \frac{\sqrt{\langle A \rangle \langle B \rangle}}{\langle A + B \rangle} \left[(\Sigma^{AB})^{1/2} - 1 \right], \quad (11)$$

where Σ^{AB} is given by Eq. (6) with $C \equiv A + B$.

A possible extension of Φ for the case of two motional variables was not discussed up to now, however it can be naturally done within the framework of the Σ^{AB} and Δ^{AB} families presented here. It is also important to note that the Φ measure extended to the study of the third moment preserves its strongly intensive properties within the model of independent sources [6]. Study of strongly intensive quantities which include 3^{rd} and higher moments of extensive quantities is beyond the scope of this paper.

Another quantity frequently used to characterize the fluctuations of particle numbers A and B was introduced in 2002 [7] as:

$$\nu_{dyn}^{AB} \equiv \frac{\langle A(A-1) \rangle}{\langle A \rangle^2} + \frac{\langle B(B-1) \rangle}{\langle B \rangle^2} - 2 \frac{\langle AB \rangle}{\langle A \rangle \langle B \rangle}. \quad (12)$$

Using Eq. (6) with $C \equiv A + B$, one easily finds the relation:

$$\nu_{dyn}^{AB} = \frac{\langle A + B \rangle}{\langle A \rangle \langle B \rangle} \left[\Sigma^{AB} - 1 \right]. \quad (13)$$

Equation (13) shows that ν_{dyn}^{AB} , similar to Σ^{AB} , is independent of fluctuations of the source number, but it decreases as $\nu_{dyn}^{AB} \propto \langle N_s \rangle^{-1}$ and, thus, it is not an intensive quantity. Note that the quantity $\langle C \rangle \nu_{dyn}^{AB}$, where C can be chosen as A , B , or $A + B$, is a strongly intensive quantity from the Σ family. Despite the fact that specific examples of the Σ^{AB} family were introduced and discussed a long time ago, the Δ^{AB} family is proposed in this paper for the first time.

IV. PROOF WITHIN THE GRAND CANONICAL ENSEMBLE

Let us now prove within the grand canonical ensemble (GCE) that the two families of quantities, Σ^{AB} and Δ^{AB} , are strongly intensive. The proof will be limited to a case of particle multiplicities, i.e., A and B will be number of particles of type A and B, respectively. The GCE partition function Ξ of the quantum gas which is a mixture of different types of particles reads:

$$\Xi = \exp \left\{ V \sum_j \eta_j d_j \int \frac{d^3 p}{(2\pi)^3} \ln [1 + \eta_j \lambda_j \exp(-\epsilon_j/T)] \right\}, \quad (14)$$

where V and T denote, respectively, the system volume and temperature, λ_j is the fugacity which is related to particle chemical potential μ_j as $\lambda_j \equiv \exp(\mu_j/T)$, d_j denotes the number of a particle internal degrees of freedom, $\epsilon_j \equiv (m_j^2 + \mathbf{p}^2)^{1/2}$ is the particle energy with m_j and \mathbf{p} being its mass and momentum, $\eta_i = -1$ for bosons, $\eta_i = 1$ for fermions, and $\eta_i = 0$ corresponds to the classical Boltzmann approximation. The GCE averages are calculated as:

$$\overline{A} = \frac{1}{\Xi} \lambda_A \frac{\partial}{\partial \lambda_A} \Xi = V \int \frac{d^3 p}{(2\pi)^3} \frac{d_A}{\lambda_A^{-1} \exp(\epsilon_A/T) + \eta_A} \equiv V n_A, \quad (15)$$

$$\overline{A^2} = \frac{1}{\Xi} \left(\lambda_A \frac{\partial}{\partial \lambda_A} \right)^2 \Xi = V^2 n_A^2 + V \int \frac{d^3 p}{(2\pi)^3} \frac{d_A \lambda_A^{-1} \exp(\epsilon_A/T)}{[\lambda_A^{-1} \exp(\epsilon_A/T) + \eta_A]^2}, \quad (16)$$

$$\overline{AB} = \frac{1}{\Xi} \lambda_A \frac{\partial}{\partial \lambda_A} \lambda_B \frac{\partial}{\partial \lambda_B} \Xi = V^2 n_A n_B, \quad (17)$$

where $n_A = \overline{A}/V$ and $n_B = \overline{B}/V$ denote the particle number densities. The corresponding expression for \overline{B} and $\overline{B^2}$ are obtained by replacing A by B in Eqs. (15-16).

For the GCE scaled variance one finds,

$$\omega_A^* \equiv \frac{\overline{A^2} - \overline{A}^2}{\overline{A}} = n_A^{-1} \int \frac{d^3 p}{(2\pi)^3} \frac{d_A \lambda_A^{-1} \exp(\epsilon_A/T)}{[\lambda_A^{-1} \exp(\epsilon_A/T) + \eta_A]^2}. \quad (18)$$

It corresponds to the particle number fluctuations at a fixed volume V . It is an intensive quantity and depends only on T and μ_A . Note that $\omega_A^* > 1$ for bosons, $\omega_A^* < 1$ for fermions, and $\omega_A^* = 1$ for classical Boltzmann particles.

We introduce now volume fluctuations assuming that local properties of the system within the GCE, i.e., the temperature and chemical potentials, are volume independent. The volume fluctuations will be described by the probability density function $F(V)$. Thus, the full averaging denoted as $\langle \dots \rangle$ will include both the GCE averaging (15-17) at a fixed volume and an averaging over the volume fluctuations:

$$\langle A \rangle = \langle V \rangle n_A, \quad \langle A^2 \rangle = \langle V^2 \rangle n_A^2 + \langle V \rangle n_A \omega_A^*, \quad \langle AB \rangle = \langle V^2 \rangle n_A n_B, \quad (19)$$

where $\langle V^k \rangle \equiv \int dV V^k F(V)$ for $k = 1, 2$. One finds,

$$\omega_A \equiv \frac{\langle A^2 \rangle - \langle A \rangle^2}{\langle A \rangle} = \omega_A^* + n_A \frac{\langle V^2 \rangle - \langle V \rangle^2}{\langle V \rangle} \equiv \omega_A^* + n_A \omega_V. \quad (20)$$

The corresponding expression for ω_B is obtained by replacing A by B in Eq. (20). Equations (20) and (5) have a similar structure. Namely, the first terms ω_A^* or ω_a^* correspond to the particle number fluctuations at a fixed volume V or fixed number of sources N_s , respectively. The second terms correspond to the contribution from the volume fluctuations in Eq. (20) and the fluctuations of the number of sources in Eq. (5).

Calculating (6-7) according to Eq. (19) with $C = A + B$ one gets:

$$\Sigma^{AB} = \frac{1}{n_A + n_B} \left[n_B \omega_A^* + n_A \omega_B^* \right], \quad \Delta^{AB} = \frac{1}{n_A + n_B} \left[n_B \omega_A^* - n_A \omega_B^* \right]. \quad (21)$$

Equation (21) proves that Σ^{AB} and Δ^{AB} are *strongly intensive* quantities as they are strictly independent of average volume $\langle V \rangle$ and its fluctuations ω_V .

Note that in the GCE there are no correlations between the number of different particle species, i.e., $\langle AB \rangle - \langle A \rangle \langle B \rangle = 0$. In the applications to hadron production in high energy collisions stable particles are detected, whereas the GCE system includes also short lived resonances which finally decay into stable particles. These resonance decays increase multiplicities of stable particles and thus change numerical values of n_A , n_B , ω_A^* , and ω_B^* . If decay products of a resonance R decay include both A and B hadrons a correlation between them appears and it can be expressed as:

$$\langle AB \rangle - \langle A \rangle \langle B \rangle = \sum_R \langle R \rangle \left[\langle AB \rangle_R - \langle A \rangle_R \langle B \rangle_R \right] \equiv \sum_R \langle R \rangle \rho_{AB}^R, \quad (22)$$

where $\langle R \rangle$ is a mean multiplicity of R and $\langle \dots \rangle_R$ means the averaging over the its all decay channels. The measure Σ^{AB} will have then the form:

$$\Sigma^{AB} = \frac{1}{n_A + n_B} \left[n_B \omega_A^* + n_A \omega_B^* - 2 \sum_R n_R \rho_{AB}^R \right], \quad (23)$$

and thus it remains a strongly intensive quantity.

V. PROPERTIES OF Σ^{AB} AND Δ^{AB} WITHIN CANONICAL AND MICRO-CANONICAL ENSEMBLES

For a large volume system in equilibrium the particle number distribution $P(A, B; V)$ in the GCE, canonical ensemble (CE), and micro-canonical ensemble (MCE) can be written in a

general form of the bi-variate normal distribution (see Refs. [8, 9]):

$$P(A, B; V) = \frac{1}{2\pi} [\omega_A^* \omega_B^* (1 - \rho_{AB}^{*2}) \bar{A} \bar{B}]^{-1/2} \times \exp \left[-\frac{1}{2(1 - \rho_{AB}^{*2})} \left(\frac{(A - \bar{A})^2}{\omega_A^* \bar{A}} - 2\rho_{AB}^* \frac{(A - \bar{A})(B - \bar{B})}{[\omega_A^* \omega_B^* \bar{A} \bar{B}]^{1/2}} + \frac{(B - \bar{B})^2}{\omega_B^* \bar{B}} \right) \right], \quad (24)$$

where $\bar{A} \equiv n_A V$ and $\bar{B} \equiv n_B V$ are mean particle numbers. Averaging at a fixed volume V is defined as ($k = 1, 2$):

$$\overline{A^k} \equiv \sum_{A,B} A^k P(A, B; V), \quad \overline{B^k} \equiv \sum_{A,B} B^k P(A, B; V), \quad \overline{AB} \equiv \sum_{A,B} AB P(A, B; V). \quad (25)$$

The straightforward calculations of (25) with the distribution function (24) give:

$$\frac{\overline{A^2} - \bar{A}^2}{\bar{A}} = \omega_A^*, \quad \frac{\overline{B^2} - \bar{B}^2}{\bar{B}} = \omega_B^*, \quad \frac{\overline{AB} - \bar{A} \bar{B}}{[\omega_A^* \omega_B^* \bar{A} \bar{B}]^{1/2}} = \rho_{AB}^*. \quad (26)$$

Equation (26) reveals the meaning of the parameters in the distribution (24) – the scaled variances ω_A^* and ω_B^* , and the correlation coefficient ρ_{AB}^* . In a mixture of relativistic ideal gases particle numbers are not conserved, and thus A and B fluctuate in all statistical ensembles. This leads to non-zero positive values of ω_A^* and ω_B^* which are approaching constant values with system volume increasing to infinity¹. Particle correlations, and thus non-zero ρ_{AB}^* , result from exact material and motional conservation laws [11]. Thus the correlation coefficient ρ_{AB}^* equals to zero in the GCE, and is non-zero in the CE and MCE. The exact conservation laws also influence the values of ω_A^* and ω_B^* . The quantities like the particle number densities do not depend on the choice of the statistical ensemble for large systems. This means thermodynamical equivalence of the statistical ensembles. Below we present results for the CE and MCE in the large volume limit in which all three statistical ensembles become thermodynamically equivalent. Let us however stress that the thermodynamical limits of the quantities (26) are different (see Ref. [11] for details) in the GCE, CE, and MCE ensembles.

We introduce now the volume fluctuations assuming that local properties of the system (e.g., temperature and conserved charge densities in the CE, or energy density and conserved charge densities in the MCE) are volume independent for large enough volumes. In this case, the distribution (24) depends on the system volume only through the average multiplicities. The full averaging reads:

$$\langle \dots \rangle \equiv \int dV F(V) \sum_{A,B} \dots P(A, B; V). \quad (27)$$

¹ The volume dependence may be different for system at the phase transition. For example, in a case of the Bose-Einstein condensation one gets, $\omega \propto V^{1/3}$ at $T = T_C$ and $\omega \propto V$ at $T < T_C$, as shown in Ref. [10].

Calculating (6-7) according to Eq. (27) with $C = A + B$ one gets:

$$\Sigma^{AB} = \frac{1}{n_A + n_B} \left[n_B \omega_A^* + n_A \omega_B^* - 2\rho_{AB}^* (n_A n_B \omega_A^* \omega_B^*)^{1/2} \right] , \quad (28)$$

$$\Delta^{AB} = \frac{1}{n_A + n_B} \left[n_B \omega_A^* - n_A \omega_B^* \right] . \quad (29)$$

Equations (28) and (29) show that Σ^{AB} and Δ^{AB} are independent of the average system volume and its fluctuations. For the CE and MCE this is valid if the volume fluctuates in a range in which all three statistical ensembles are thermodynamically equivalent.

A unique determination of five intensive quantities, n_A , n_B , ω_A^* , ω_B^* , and ρ_{AB}^* , from measurements of $\langle A \rangle$, $\langle B \rangle$, $\langle A^2 \rangle$, $\langle B^2 \rangle$, and $\langle AB \rangle$, is impossible as $\langle V \rangle$ and ω_V are in general unknown. The average particle multiplicities are given by $\langle A \rangle = n_A \langle V \rangle$ and $\langle B \rangle = n_B \langle V \rangle$. Therefore, only the ratio of particle number densities, $r_{AB} \equiv \langle A \rangle / \langle B \rangle = n_A / n_B$, can be found from the measurements of $\langle A \rangle$ and $\langle B \rangle$. The three strongly intensive quantities, r_{AB} , Σ^{AB} , and Δ^{AB} , allow a unique determination of ω_A^* and ω_B^* in the GCE, if resonance decay effects are absent. In this case there are no correlation between A and B at fixed volume, and from Eq. (21) one finds:

$$\omega_A^* = 2(1 + r_{AB}) [\Sigma^{AB} + \Delta^{AB}] , \quad \omega_B^* = 2(1 + r_{AB}^{-1}) [\Sigma^{AB} - \Delta^{AB}] . \quad (30)$$

However, if $\rho_{AB}^* \neq 0$, as in the CE and MCE, or including correlations due to resonance decays in the GCE, even the knowledge of all strongly intensive quantities is not sufficient to reconstruct ω_A^* , ω_B^* , and ρ_{AB}^* in a unique way.

VI. SUMMARY

In summary, in this paper we consider two families of strongly intensive quantities Σ^{AB} and Δ^{AB} which characterize fluctuations of system properties. While specific measures from the Σ^{AB} family were introduced already in 1992 [2], the Δ^{AB} family is proposed in this paper for the first time. We prove within the grand canonical ensemble that both Σ^{AB} and Δ^{AB} quantities are strictly independent of volume and volume fluctuations. In the canonical and micro-canonical ensembles they are approximately independent of volume and volume fluctuations for large enough systems. Note that the Φ and ν_{dyn}^{AB} measures, which can be expressed in terms of Σ^{AB} , have already been used successfully to study transverse momentum and particle ratio fluctuations (see, e.g., Refs. [12, 13] and references therein). We hope that the results presented in this paper will be useful in further analysis of fluctuations of hadron production properties in collisions of relativistic particles.

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- [1] For recent review see W. Florkowski, *Phenomenology of Ultra-Relativistic Heavy Ion Collisions*, World Scientific , 2009.
- [2] M. Gaździcki and S. Mrówczyński, Z. Phys. C **54**, 127 (1992).
- [3] F. Liu, A. Tai, M. Gazdzicki *et al.*, Eur. Phys. J. C **8**, 649-654 (1999).
- [4] M. Gaździcki, Euro. Phys. J. C **8**, 131 (1999).
- [5] S. Mrówczyński, Phys. Lett. B **459**, 13 (1999).
- [6] S. Mrówczyński, Phys. Lett. B **465**, 8 (1999).
- [7] C. Pruneau, S. Gavin, S. Voloshin, Phys. Rev. C **66**, 044904 (2002).
- [8] M. Hauer, V.V. Begun, and M.I. Gorenstein, Eur. Phys. J. C **58**, 83 (2008).
- [9] V.V. Begun, M. Gaździcki, and M.I. Gorenstein, Phys. Rev. C **80**, 064903 (2009).
- [10] V. V. Begun and M. I. Gorenstein, Phys. Lett. B **653**, 190 (2007); Phys. Rev. C **77**, 064903 (2008).
- [11] V. V. Begun, M. Gaździcki, M. I. Gorenstein, and O. S. Zozulya, Phys. Rev. C **70**, 034901 (2004); V.V. Begun, M.I. Gorenstein, M. Hauer, V.P. Konchakovski, and O.S. Zozulya, Phys. Rev. C **74**, 044903 (2006); V.V. Begun, M. Gaździcki, M.I. Gorenstein, M. Hauer, V.P. Konchakovski, and B. Lungwitz, Phys. Rev. C **76**, 024902 (2007).
- [12] T. Anticic *et al.* [NA49 Collaboration], Phys. Rev. C **79**, 044904 (2009); C. Alt *et al.* [NA49 Collaboration], Phys. Rev. C **70**, 064903 (2004).
- [13] V. Koch and T. Schuster, Phys. Rev. C **81**, 034910 (2010).